**Math 22**
**Quiz 5**
**Fri Nov 9, 2012**

**NAME YOU ASKED TO BE CALLED IN CLASS:** ____________________

**SCORE: _____ / 35 POINTS**

Find and prove an explicit formula for the sequence defined recursively by **SCORE: _____ / 14 PTS**

$$a_n = 5a_{n-1} - 3 \text{ for all } n \in Z^+, \quad a_0 = 2$$

$a_1 = 5 \cdot 2 - 3$

$a_2 = 5(5 \cdot 2 - 3) - 3 = 5^2 \cdot 2 - 5 \cdot 3 - 3$

$a_3 = 5(5^2 \cdot 2 - 5 \cdot 3 - 3) - 3 = 5^3 \cdot 2 - 5^2 \cdot 3 - 5 \cdot 3 - 3$ (2)

$a_n = 5^n \cdot 2 - \dfrac{3(5^n - 1)}{5-1}$ (4) OR

$= 2 \cdot 5^n - \frac{3}{4}(5^n - 1)$ OR

$= \frac{5}{4} \cdot 5^n + \frac{3}{4}$

$= \dfrac{5^{n+1} + 3}{4}$

IF YOU DID NOT SIMPLIFY $a_n$ BEFORE THE INDUCTION PROOF, YOUR STEPS BELOW WILL ALL LOOK DIFFERENT, SO TALK TO ME

PROOF BY MI:

BASIS STEP: $a_0 = \dfrac{5^{0+1} + 3}{4} = \dfrac{5+3}{4} = 2$ ✓ (1)

INDUCTIVE STEP: ASSUME $a_k = \dfrac{5^{k+1} + 3}{4}$ (1) FOR SOME PARTICULAR BUT ARBITRARY INTEGER $k$ (1) $\geq 0$ (1)

$a_{k+1} = 5a_k - 3$

$= 5\left(\dfrac{5^{k+1} + 3}{4}\right) - 3$ (1)

$= \dfrac{5^{k+2} + 15 - 12}{4}$ (1)

$= \dfrac{5^{k+2} + 3}{4}$ (1)

BY MI, $a_n = \dfrac{5^{n+1} + 3}{4}$ FOR ALL $n \in \mathbb{Z}^*$ (1)

Fill in the blanks by writing the **symbolic** translations. Do not use $\sim, \cup, \cap, -$ or $^C$ in your answers. **SCORE: _____ / 6 PTS**

(Assume that $x$ is an element, and $A, B, C$ are subsets of universal set $U$.)

[a] $x \in A^C \cap B$ if and only if $x \notin A \wedge x \in B$ (2)

[b] $A \not\subseteq B$ if and only if $\exists x \in A : x \notin B$ (2)

[c] $x \notin A - B$ if and only if $x \notin A \vee x \in B$ (2)

SUBTRACT 1 POINT FOR EACH $\forall, \exists, \{\}$ THAT'S NOT SUPPOSED TO BE THERE

---

One of the following statements is true and one is false. **SCORE: _____ / 15 PTS**

For the true statement, write a formal **element proof**. **DO NOT USE SET ALGEBRA.**

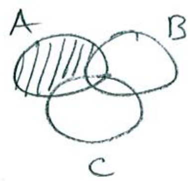
For the false statement, find a **minimal** counterexample. **Demonstrate clearly that your counterexample disproves the false statement.**

[a] For all sets $A, B, C \subseteq U$, $A-(B-C) = (A-B)-C$.

[b] For all sets $A, B, C \subseteq U$, if $A \subseteq B - C$, then $A \cap C = \varnothing$.

[a] IS FALSE.

$A-(B-C)$



LET $A = C = \{1\}$, (1) $B = \varnothing$ (1)

$B - C = \varnothing$

(1) $A-(B-C) = \{1\}$

$A - B = \{1\}$

(1) $(A-B)-C = \varnothing$

NOT EQUAL

[b] IS TRUE

PROOF:

LET $A, B, C \subseteq U$ ($\frac{1}{2}$) SUCH THAT $A \subseteq B - C$ (1)

SUPPOSE THAT $A \cap C \neq \varnothing$ (1)

SO, THERE EXISTS AN ELEMENT $x \in A \cap C$ (1)

BY DEF'N OF $\varnothing$ ($\frac{1}{2}$)

SO, $x \in A$ AND $x \in C$ (1) BY DEF'N OF $\cap$ (1)

SO (1) $x \in B - C$ BY DEF'N OF $\subseteq$ ($\frac{1}{2}$)

SO (1) $x \in B$ AND $x \notin C$ BY DEF'N OF $-$ (1)

SO, $x \in C$ AND $x \notin C$ (CONTRADICTION) (1)

SO, $A \cap C = \varnothing$ ($\frac{1}{2}$)

NOTE: ORDER OF STATEMENTS IN THIS PROOF IS CRUCIAL